Critique of Robert Taylor's "A Game-Theoretic Model of Gun Control"

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1 Introduction

In "A Game Theoretic Model of Gun Control" (Taylor 1995), Robert Taylor attempts to model the effects of gun control on the utility of the innocent citizen, the potential victim of violent crime. Specifically, he sets out to provide theoretical support for Polsby's (1993) contention, published in the Libertarian magazine *Reason*, that "even if univesral disarmament is optimal, gun control measures that merely move us in that direction may *reduce* social welfare." (Taylor 1995: 270, italics mine). Empirical studies having been inconclusive, he argues, a more theoretical approach and a simplified model may turn up insights previously passed over.

Taylor, then, begins with a well-defined hypothesis to test, and the basic outline of an experiment with which to test it: he intends to develop a normative game-theoretical model of gun control which demonstrates contradictions to two implicit assumptions that "most arguments for gun control make": that "universal disarmament is an optimal state," and that "gun control, by moving us toward this optimal state, will necessarily increase welfare" (270), and which can be used to determine the direction which gun-control policy should take. Unlike Brown and Boswell, Taylor chooses to start from scratch on his model.

2 Taylor's Games

Taylor borrows further from the hard sciences in his methodology: first, generate a simple model based on acknowledged assumptions, and then from that develop a complex model which accounts for as many of the assumptions made in the first model as is feasible. This approach is somewhat foreign to sociology, with its love of grand theories and distaste for intentional simplification, but is well-established in hard sciences (*cf.* the hoary old physicist-biologist joke which begins, and often ends, "Imagine a perfectly spherical elephant in a vacuum..."), and for the purpose of making policy, being able to examine the effects of the element being controlled by policy in isolation can be useful—assuming that the variables in question can, in fact, be isolated. At the same time, he assumes the burden not only to ensure that his conclusions logically follow from the model, but that the model itself is sound and consistent.

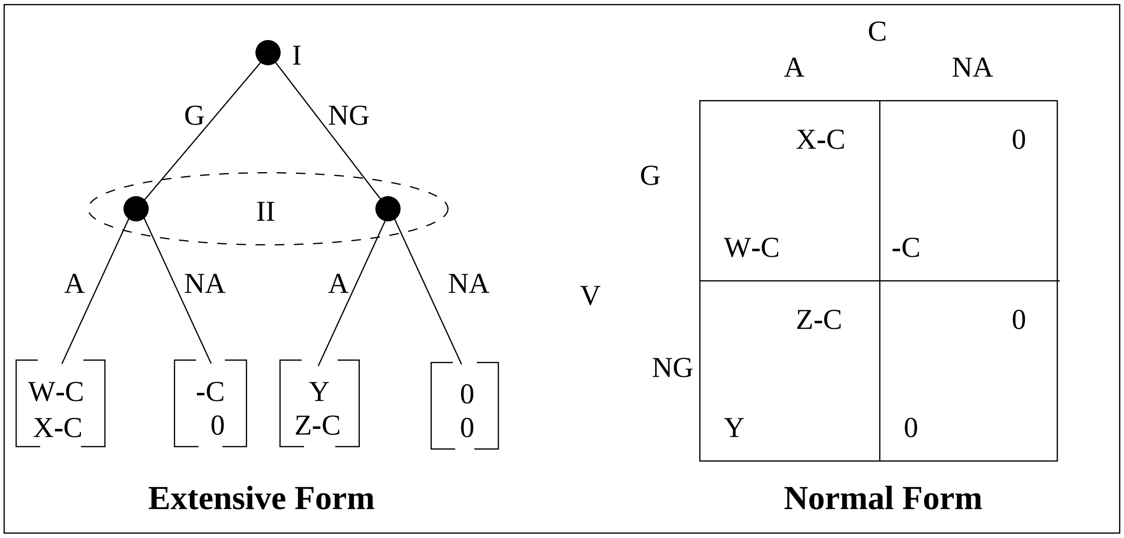

Figure 1: Taylor's basic gun-control game.

2.1 The Basic Model

Taylor's basic model is a one-shot simultaneous-move game played between Player I, a potential victim who may or may not be armed, and Player II, a criminal who may or may not attack Player I. The game is played as illustrated in Figure 1.

Player I and Player II move simultaneously, thus it is also a game of imperfect information (Binmore 1992: 501); the extensive form shown in Figure 1(a) could just as easily show the criminal as having the first move. Player I and Player II are assumed to be risk-neutral to simplify the calculation of payoffs. C in Figure 1 represents the cost of obtaining a gun, and is the sum of a constant, C , representing the price of a gun, and a variable, R, representing additional costs imposed by gun-control measures (271). Most importantly, varying R changes the nature of game's Nash equilibria; the game is not a particular type (Prisoner's Dilemma, Chicken, etc.) but one whose equilibria shift according to the R of a particular policy choice. The concern, then, is not so much a description or prediction of the player's behaviour as it is the observation of *changes* in behaviour at different valuse of R. [1] R is assumed to be identical for Player I and Player II.

Taylor expresses the preference relations of the game as $Z \quad 0 \quad X \quad W \quad Y$ where Z and X are the criminal's payoffs from attacking an armed and an unarrmed victim,

[1] For a thorough treatment of altering the nature of a game via external variables, see Heckathorn (1996).

respectively, and W and Y are the armed and unarmed victim's payoffs from an attack (271). This preference relation should immediately warn us that something is amiss; the relations $(X \quad W), (Z \quad W), (X \quad Y)$, and $(Z \quad Y)$ assume transferable utility, which except when explicitly dealing with a measurement of utility valued equally by both parties (for example, money) is an invalid assumption (Binmore 1992: 115). Luckily for us (or for Taylor!) the preferences which assume transferable utility are not compared with each other when payoffs are determined. The oversight does illustrate how easily errors can slip into otherwise-straightforward models.

More curious is $(0 \quad X)$, which suggests that a criminal will prefer not attacking an armed victim to being evenly matched with him, which is nonintuitive given that Taylor explictly notes that the criminal in the encounter has the "advantage of surprise" (271). He uses this "advantage of surprise" in order to support $(X \quad W)$, though, so discarding that relation allows us to discard the imprecise "advantage of surprise", leaving a criminal preferring not attacking to being in a perflectly-matched fight, which, while not the only possibility, is at least a possibility. Having ensured that the surface imperfections in preference relations do not disturb the model, we can see where the model leads.

2.1.1 Taylor's conclusions

Taylor correctly perceives that sufficiently large values of C make $[A, NG]$ and $[NA, NG]$ Nash equilibria (273). In order to exclude these trivial cases, the range of C is restricted to values low enough to force mixed strategies.

In mixed strategies, Player I has a certain probability of being armed, and Player II has a certain probability of attacking; the Nash equilibrium is the point at which the respective probabilities balance out such that a player is indifferent about choosing one strategy over the other. Taylor's calculations of payoffs are technically accurate and merit no discussion here other than to note the payoff to Player I, which is the dependent variable which we wish to maximize in determining gun-control policy, that is, choosing a value for R. Thay payoff is $CY/(W \quad Y)$.

The derivative of that with respect to R is $Y/(W \quad Y)$ which is negative, leading to a startling observation: *any increase in the cost of obtaining a gun reduces the payoff for the victim.* In other words, the model is consistent with Polsby's conclusions. (In fact, the payoff function for Player I is discontinuous; when it no longer pays off for Player II to use a gun (*ie.*, when $Z \quad C$), Player I's payoff raises to 0 in the utopian outcome $[NG, NA]$. Taylor, assuming universal disarmament unrealistic, discards that outcome (275); while not explicitly true to the model, this seems a reasonable restriction, especially in light of his comment that the Government cannot prevent someone from getting something they want, they can only make it more difficult (272).)

Taylor's primary conclusion from the simple model is that any attempt at gun control will reduce social welfare (275); "gun control tends to disarm victims and encourage criminal predation." Within the model this appears unequivocal. However, he concludes further that a move from the mixed-strategy Nash equilibrium to $[G, NA]$ would increase social welfare (277). He begins his explanation, "Suppose that criminals could observe the armament decisions of individuals". Certainly, then, NG would be strongly dominated, and thus so would A, leaving $[G, NA]$; but that is an different game than the one analyzed, being no longer simultaneous-move nor of imperfect information.

It would also appear to require every man, woman and child to carry a gun and be able to use it. Since the minimum cost of having a gun is $C\ \ , C\ \ > 0$, this introduces a free-rider problem where one potential victim *not* being armed would diverge from $[G, NA]$, bringing us back into mixed-strategy equilibriums, but with a different and considerably more complex model. This *non sequitur* regarding $[G, NA]$ resides outside the model and is not supported by it; it appears to be the result of Taylor's political viewpoints regarding the concealed carrying of firearms (277) leading to creative interpretations of the boundaries of the model.

(Note also that $[G, NA]$ may not be an equilibrium if we factor in the "advantage of surprise" that Taylor gives the attacker, which we—but not Taylor—discarded earlier; if evenly-matched armed encounters lead to a high enough payoff for the attacker, the equilibrium could just as easily be $[NG, A]$: where attackers have the upper hand in an attack, they will always attack, so victims who lose either way will avoid the cost of obtaining a gun and go unarmed. But this is as external to the model as is Taylor's universal civil armnament outcome, $[G, NA]$, and should only serve to demonstrate the wildly erroneous conclusions which can result from the introduction of imprecise concepts—in this case, the advantage of surprise and observed armnament decisions.)

2.2 The Complex Model

In the second part of his article, Taylor attempts to account for some of the assumptions made in his basic model. In terms of method, this is practical; iteratively refining the model and observing the effects of the refinement on the outcome appears to be a particularly productive way of not only finding results but determining as soon as possible if the refinements provide useful results. Practically, however, an increase in complexity means an increase in the potential for imprecision and faulty assumptions, which when trying

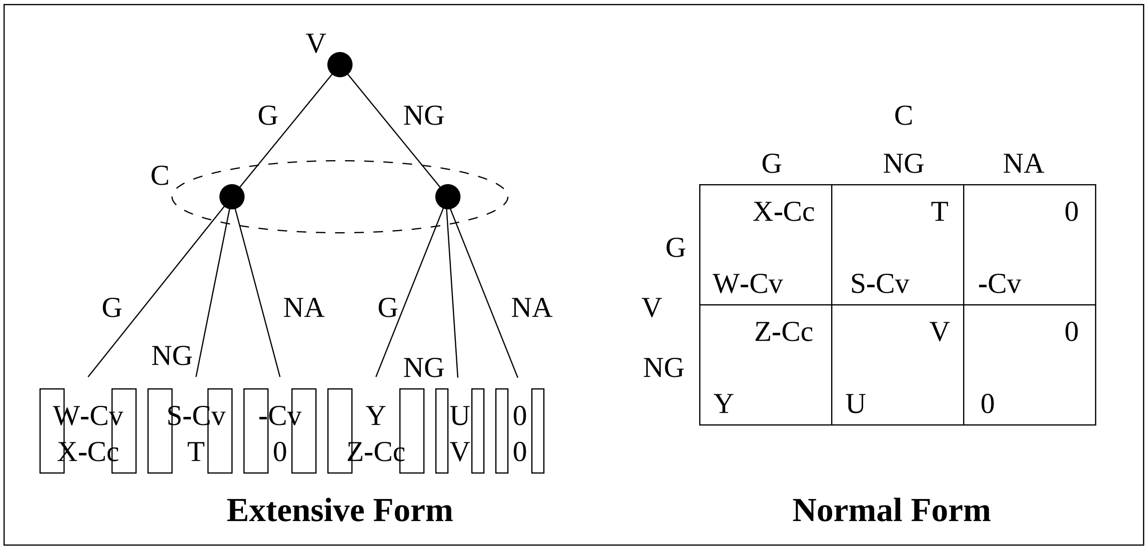

Figure 2: Taylor's complex gun-control game.

to both develop and prove a model *and* analyze its implications within one article are increasingly difficult to avoid.

The second model makes two important refinements on the basic one. First, Player II has a third option, NG, to attack without a gun; second, criminals can be targeted by gun-control measures, such that the cost of obtaining a gun for Player II (C_c) is higher than that for Player I (C_v). The game is illustrated in Figure 2.

Again, Taylor's preference relation suggests transferable utils, which is not the case, and again the relations which suggest that have no effect on the outcome. Player I's preferences are $0 \quad S \quad U \quad W \quad Y$, where S and U are an armed and unarmed victim's payoffs from an unarmed attack, and W and Y from an armed attack, respectively (280). These seem unobjectionable. Player II's preferences are $Z \quad V \quad 0 \quad X \quad T$, where Z and V are the armed and unarmed criminal's payoffs for attacking an unarmed victim, and X and T those for attacking an armed victim (278). Taylor again appears to order preferences based on the "advantage of surprise"; even if we discard that along with the transferable utils, we end up with questionable relations: $V \quad 0 \quad X$ suggests that an evenly-matched unarmed fight produces a higher payoff than an evenly-matched armed fight, for which Taylor gives no explanation. (Even if $V \quad 0 \quad X$ was chosen arbitrarily, explicitly saying so reduces the need for speculation that potentially important aspects of the model received insufficient attention.)

2.2.1 Equilibria

For reasons as above, pure strategies are dropped from consideration (280). This model ends up with two distinct mixed-strategy Nash equilibria; when R is low enough to make gun use efficient, Player II chooses between G and NA, and when R is high enough to make gun use inefficient, Player II chooses between NG and NA (281). Further, the slope of the plot of Player I's payoff vs. R decreases (282). In other words, by introducing NG and by using a higher R for Player II (criminal) than Player I (victim), we see the following developments: Player II will not attack with a gun when it means a sure loss to do so; Player II will not attack without a gun when it means a sure loss to do so; and Player I's welfare will not decrease as rapidly with increases in gun-control measures.

2.2.2 Taylor's Conclusions

While Taylor seems to present these as important changes (282), they appear a little tautological; furthermore, the payoff function with respect to R still has a discontinuity at the point at which Player II moves from G to NG as his "attack" strategy. Inexplicably, Taylor suggests that "modest gun control actually increases social welfare" (283); the point at which it does so is the point at which all criminals are disarmed. It is not clear why this scenario which was unattainable in the first model is now attainable; since nothing has changed to allow it, logic would suggest that the condition mentioned by Taylor regarding Government inefficacy still applies. This is to say, the point at which G is strongly dominated in the second model is the same as the point at which G is strongly dominated in the first, and that point will not be reached for the same reasons.

Contrary to his conclusions, the refined model appears to uphold the conclusion of the basic model—that since disarmament cannot be obtained, any gun control measures will reduce social welfare.

3 Evaluation

With a well-constructed basic model, Taylor has discovered an aspect of gun-control policy which goes against common-sense perceptions; that alone makes his an important study and a practical application of game theory. His attempts at introducing a more realistic model indicated on their own that the conclusions from the basic model continue to stand as real-world complications find their way in; although his stated conclusions and those which the enhanced model presented diverged somewhat, the findings from the basic model still continued to stand.

That being said, the focus of this paper was gun-control *policy*—is it possible to go from this model to a workable policy? In this particular case, that doesn't appear to be possible. The model is too simple; as Taylor points out, something as simple as noting that criminals can attack other criminals (287) complicates the question of who needs to own a gun and who will be doing the attacking enough to make this model inaccurate. He also notes that the assumption of risk-neutrality is probably inaccurate (287); if the players in the game have different risk assessments, mixed strategies change, and the patterns observed above may no longer appear.

As well, the assumption made at the beginning of the paper—that "most arguments" about gun control claim that by moving closer to universal disarmament, social welfare increases—does not seem to be supported by the claims of gun-control advocates, who concentrate as much on accidental injury, suicide prevention, and gun culture as they do crime control (Spitzer 1998: 65; Carter 1997: 105). Paralleling that, the number of firearm deaths by suicide remains slightly *above* those from homicide (National Center for Injury Prevention and Control 2001: 2). Between the questionable premise of the study and the creative policy recommendations that resulted from it, it seems as though Taylor has spent much of his effort attacking a straw man.

From this two further observations on the utility of game theory in policy development can be drawn. First, that by providing simplified models with well-defined preferences and measurable outcomes, game-theoretic models provide the tools to discover relationships between policy and performance which might otherwise escape (empirical) study, such as the declining-welfare effects measured here; but in doing so, such models are necessarily unable to stand on their own as justification for policy. Lastly, that reaching a preliminary conclusion based on a simplified model may be considered a 'second-class' result in sociological or policy research in which real-world answers are the immediate goal. Fields in which that remains the case need to look to economics, statistical mathematics, and the hard sciences to see the utility of preliminary and abstract work; one can only hope that little work of value but too straightforward for publication falls through the cracks before that utility is noticed.

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